Early warning signals in theoretical and real-world social contagion systems

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Consumer Fad Case Study

To characterize CSD in a real-world consumer system, a data set for a faddish product line was analyzed using the R package spaero. The product in question was a handmade crocheted hat modeled off of the hair of Cabbage Patch Kids dolls, sold by the shop The Lillie Pad on the online marketplace Etsy. It was offered on the site for approximately two years (June 2011 - June 2013), exploding in popularity, landing product profiles by the Huffington Post, Today, ABC News, Perez Hilton, Daily Mail, and others.

The time series was divided into four classes: Susceptible, Infected, Buzz, and Recovered (Fig. 1).

This SIBR model assumes transmissibility of both active adopters/advocates (analogous to rumor-spreading) and "buzzers": those who were recently active participants in its direct advocacy, or who are passive participants (onlookers, listeners, inquirers) observed by susceptibles.

The SIBR system of ordinary differential equations:

\[
\frac{dS}{dt} = -S(\beta I + \delta B + \varepsilon + \beta) + \alpha S \\
\frac{dI}{dt} = S(\beta I + \delta B + \varepsilon + \beta) - \gamma - \alpha I + r \beta I \\
\frac{dB}{dt} = (1 - \varepsilon) I - \alpha B + \beta B \\
\frac{dR}{dt} = \alpha B \\
\end{align}
\]

The SIBR model integrates contagion ubiquity over time with the cumulative factor \( \varepsilon \). A social contagion becomes more passively transmissible (\( \beta_L \)) as the number of infecteds and buzzers increases over time. The cumulative factor \( \alpha \) is directly multiplied to \( \beta_L \) as a means of increasing the force of infection with increasing popularity. Unlike conventional hinge models of \( I_0 \), the SIBR model has a built-in positive feedback loop that is capable of pushing a non-critical system to its critical threshold.

\[
\lambda = \beta_L I + \delta B + \varepsilon + \beta \\
\frac{dI}{dt} = \lambda I - \gamma I - \alpha I \\
\frac{dB}{dt} = (1 - \varepsilon) I - \alpha B + \beta B \\
\frac{dR}{dt} = \alpha B \\
\end{align}
\]

The force of infection (Eq. 6) results from direct endorsement, ubiquity, and spontaneous exposure. The cumulative effect-dependent reproduction number (Eq. 7) was derived using the next generation matrix method.3 The critical threshold occurs when \( R_{cr} = \frac{\alpha}{\beta} \) and can be solved for \( \varepsilon_{\text{opt}} \) with a given set of parameters (Eq. 8).

Future directions: Conduct performance analysis of EWS with changing transmissibility

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References


Cabbage patch hat photo courtesy of The Lillie Pad.